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Relativistic corrections to Υ exclusive decay into double S-wave charmonia

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Abstract

Within the framework of nonrelativisitic QCD factorization formalism, we present the next-to-leading-order relativistic corrections to Υ exclusive decay into η_c plus J/ψ. The double charmonia can be produced through several immediate channels, i.e., $\Upsilon \to g^* g^* g^* \to \eta_c + J/\psi$, $\Upsilon \to g^* g^* \gamma^* \to \eta_c + J/\psi$, and $\Upsilon \to \gamma^* \to \eta_c + J/\psi$. The amplitudes of these three channels are obtained accurate up to $\mathcal{O}(\alpha_s^3 v^2)$, $\mathcal{O}(\alpha\alpha_s^2 v^2)$, and $\mathcal{O}(\alpha\alpha_s^2 v^2)$, respectively, where v indicates the typical heavy quark velocity in bottomonium and/or charmonium rest frame. The decay rates are also presented. We find that the next-to-leading-order relativistic corrections to the short-distance coefficients as well as the decay rates are both significant and negative, especially for the corrections from bottomonium. More seriously, the decay rates are even brought into negative by including the relativistic corrections, which indicates the poor convergence for the velocity expansion in this kind of process. Detailed analysis is given in the paper.

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I. INTRODUCTION

Nonrelativistic QCD (NRQCD) [1] is a powerful and successful effective field theory in describing the quarkonium production and decay. The processes of bottomonium exclusive decay into double charmonia are interesting channels to investigate quarkonium, which have been extensively studied both in theory and experiment [2, 3]. The η_b decay into double J/ψ was studied in Refs. [4–8], and the radiative and relativistic corrections to this process were investigated subsequently in [9–11]. The P-wave bottomonium exclusive decay into double charmonia was also studied in Refs. [12, 13]. The relativistic corrections to these processes were obtained in Refs. [11, 14], and the radiative corrections to $\chi_{bJ} \to J/\psi J/\psi$ were calculated recently [15]. For the case of Υ, the $\Upsilon \to \eta_c J/\psi$ and $\Upsilon \to J/\psi \chi_{cJ}$ were separately studied in Ref. [16] and Ref. [17].

Comparing with other bottomonium exclusive decay processes mentioned above, $\Upsilon \to \eta_c J/\psi$ has some distinctive features. First, there is no tree-level pure QCD Feynman diagram. Second, this process is further suppressed by a helicity selection rule due to $m_b \gg m_c$. As a consequence, one may expect that the decay rate is relatively small [16]. Nevertheless, it is still meaningful to investigate this process based on the following reasons. First, there is little study of the relativistic corrections to the quarkonium decay and production processes with the vanishing tree-level Feynman diagram, so it is theoretically interesting to study the relativistic corrections and the convergence of the velocity expansion for these processes. Second, it has been known for a long time that the relativistic corrections to $J/\psi \to$ LH and $J/\psi \to 3\gamma$ are both significant and negative [18], which even surpasses the contributions at leading order. The study in Ref. [19] indicated that the $\mathcal{O}(\alpha_s v^2)$ corrections to $J/\psi \to 3\gamma$ are also huge; meanwhile, the study in Refs. [20, 21] indicates that the relativistic corrections to $\Upsilon \to g^* gg \to c\bar{c}gg$ are extraordinarily large, despite the small $\mathcal{O}(v^2)$ NRQCD matrix element for Υ. The common feature for these processes is that Υ first annihilates into three gluons (photons). For the process $\Upsilon \to \eta_c J/\psi$, the main contributions come from the immediate channel $\Upsilon \to g^* g^* g^* \to \eta_c J/\psi$ [16]. As a consequence, it is compelling to know whether large relativistic corrections are also appearing in $\Upsilon \to g^* g^* g^*$. Third, there has been a longstanding "ρ-π" puzzle in quarkonium decay [22, 23]. The structure of Feynman diagrams for $\psi \to \rho\pi$ is rather similar to $\Upsilon \to \eta_c J/\psi$, so one may wonder whether the puzzle originates from the relativistic corrections for J/ψ. Based on these reasons, we will investigate the relativistic corrections to $\Upsilon \to \eta_c J/\psi$ from both the charmonia and the bottomonium.

Up to the next-to-leading order in velocity expansion, there are three channels which contribute, $\Upsilon \to g^* g^* g^* \to \eta_c J/\psi$, $\Upsilon \to g^* g^* \gamma^* \to \eta_c J/\psi$, and $\Upsilon \to \gamma^* \to \eta_c J/\psi$. The leading-order (LO) amplitudes for the three channels in powers of α_s and α scale as α_s^3, $\alpha\alpha_s^2$, and $\alpha\alpha_s$, respectively. Though the second channel is suppressed by a factor α/α_s relative to the first channel, there is a fragmentation enhancement ($\gamma^* \to J/\psi$) in $\Upsilon \to g^* g^* \gamma^* \to \eta_c J/\psi$; as a result, we also consider this channel with J/ψ produced by a photon in our calculation. Moreover, since $\Upsilon \to \gamma^* \to \eta_c J/\psi$ is of a similar Feynman diagram structure as $e^+e^- \to \eta_c J/\psi$, the radiative corrections to which were found to be large [24, 25], the calculation in this channel will be accurate up to $\mathcal{O}(\alpha\alpha_s^2 v^2)$. [1]

[1] We do not consider the pure QED contributions in $\Upsilon \to \gamma^* \to \eta_c J/\psi$, due to the fact the pure QED contributions are less than a third of other contributions [26], and what is more, $\Upsilon \to \gamma^* \to \eta_c J/\psi$ contributes merely a small fraction in $\Upsilon \to \eta_c J/\psi$ decay [16].

The rest of the paper is organized as follows. In Sec. II, we describe the NRQCD formula for Υ exclusive decay into double charmonia and present the kinematic descriptions and required techniques. The short-distance coefficients (SDCs) are obtained in detail in Sec. III. Section IV is devoted to numerical predictions. Discussions and summary are also presented in this section.

II. NRQCD FORMULA AND KINEMATIC DESCRIPTIONS

According to Lorentz invariance, we are able to factorize Υ exclusive decay into double S-wave charmonia as

$$\mathcal{A} \equiv \langle \eta_c J/\psi(\epsilon_2) | \Upsilon(\epsilon_1) \rangle = i\epsilon^{\mu\nu\alpha\beta} \epsilon_{1\mu} \epsilon_{2\nu}^{*} P_{1\alpha} P_{2\beta} A, \tag{1}$$

where ϵ tensor indicates the Levi-Civita symbol, ϵ_1 and ϵ_2 represent the polarization vectors of the initial state Υ and final state J/ψ, P_1, and P_2 denote the momenta of η_c and J/ψ, respectively.

As mentioned in Ref. [11], the exclusive decay of a bottomonium into double charmonia involves the annihilation of a $b\bar{b}$ pair followed by the creation of two pairs of $c\bar{c}$. One may guess that the generalization of the NRQCD factorization for the electromagnetic decay or light-hadronic decay into this exclusive mode is possible. So according to the NRQCD factorization formalism, the Lorentz scalar A can be factorized as

$$A = \sqrt{2m_{\eta_c} 2m_{J/\psi}} \sqrt{\langle \mathcal{O} \rangle_{\eta_c} \langle \mathcal{O} \rangle_{J/\psi} \langle \mathcal{O} \rangle_{\Upsilon}} \; c_0 \left(1 + c_{2,1} \langle v^2 \rangle_{\Upsilon} + c_{2,2} \langle v^2 \rangle_{\eta_c} + c_{2,3} \langle v^2 \rangle_{J/\psi} + \mathcal{O}(v^4)\right), \tag{2}$$

where c_0 and c_2 correspond to the SDCs, which can be calculated perturbatively in power of α_s. For simplicity, we define shortcuts for the matrix elements

$$\begin{aligned} \langle \mathcal{O} \rangle_{\Upsilon} &= |\langle 0 | \chi^{\dagger} \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}_1^{*} \psi | \Upsilon(\epsilon_1) \rangle|^2, & \text{(3a)} \\ \langle \mathcal{O} \rangle_{\eta_c} &= |\langle \eta_c | \psi^{\dagger} \chi | 0 \rangle|^2, & \text{(3b)} \\ \langle \mathcal{O} \rangle_{J/\psi} &= |\langle J/\psi(\epsilon_2) | \psi^{\dagger} \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}_2 \chi | 0 \rangle|^2, & \text{(3c)} \end{aligned}$$

and

$$\begin{aligned} \langle v^2 \rangle_{\Upsilon} &= \frac{\langle 0 | \chi^{\dagger} \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}_1^{*} \big(-\frac{i}{2} \overleftrightarrow{\mathbf{D}} \big)^2 \psi | \Upsilon(\epsilon_1) \rangle}{m_b^2 \langle 0 | \chi^{\dagger} \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}_1^{*} \psi | \Upsilon(\epsilon_1) \rangle}, & \text{(4a)} \\ \langle v^2 \rangle_{\eta_c} &= \frac{\langle \eta_c | \psi^{\dagger} \big(-\frac{i}{2} \overleftrightarrow{\mathbf{D}} \big)^2 \chi | 0 \rangle}{m_c^2 \langle \eta_c | \psi^{\dagger} \chi | 0 \rangle}, & \text{(4b)} \\ \langle v^2 \rangle_{J/\psi} &= \frac{\langle J/\psi(\epsilon_2) | \psi^{\dagger} \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}_2 \big(-\frac{i}{2} \overleftrightarrow{\mathbf{D}} \big)^2 \chi | 0 \rangle}{m_c^2 \langle J/\psi(\epsilon_2) | \psi^{\dagger} \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}_2 \chi | 0 \rangle}. & \text{(4c)} \end{aligned}$$

In Eq. (2), the bottomonium is normalized nonrelativistically on both sides; in contrast, the charmonia are normalized relativistically on the left-hand side, but nonrelativistically on the right-hand side. The explicit factor $\sqrt{2m_{\eta_c} 2m_{J/\psi}}$ is responsible for the discrepancy.

The SDCs are free of nonperturbative effects and can be obtained through the matching technique. To calculate the SDCs, we are allowed to substitute the quarkonium states to free heavy quark pairs of the same quantum numbers as the corresponding hadrons. Now A and

the matrix elements in (2) can be calculated perturbatively; therefore, the SDCs are readily derived. The spin-singlet and -triplet states of a free heavy quark pair can be extracted by employing the spin projectors in Ref. [27]:

$$\Pi_3(p_0,\bar{p}_0) = \frac{1}{8\sqrt{2N_c}E(q_0)^2[E(q_0)+m_b]}(\not{p}_0+m_b)[\not{P}_0+2E(q_0)]\not{\epsilon}_1(\not{\bar{p}}_0-m_b), \tag{5a}$$

$$\overline{\Pi}_1(p_1,\bar{p}_1) = \frac{1}{4\sqrt{2N_c}E(q_1)[E(q_1)+m_c]}(\not{\bar{p}}_1-m_c)\,\gamma_5[\not{P}_1+2E(q_1)](\not{p}_1+m_c), \tag{5b}$$

$$\overline{\Pi}_3(p_2,\bar{p}_2) = -\frac{1}{4\sqrt{2N_c}E(q_2)[E(q_2)+m_c]}(\not{\bar{p}}_2-m_c)\,\not{\epsilon}_2^*[\not{P}_2+2E(q_2)](\not{p}_2+m_c), \tag{5c}$$

where a color-singlet factor $\frac{1}{\sqrt{N_c}}$ is included, and $E(q_i)=\sqrt{m^2+\boldsymbol{q}_i^2}$ with $m=m_b$ for $i=0$ and $m=m_c$ for $i=1,2$. In (5), we take P_i and q_i to be the total momenta and half of the relative momenta of the quark pairs in the quarkonia, where $i=0$ denotes Υ, $i=1$ denotes η_c, and $i=2$ denotes J/ψ. Therefore, the momenta of the corresponding quarks p_i and antiquarks $\bar{p}_i$ in the hadrons are expressed as

$$p_i = \frac{1}{2}P_i + q_i, \tag{6a}$$

$$\bar{p}_i = \frac{1}{2}P_i - q_i. \tag{6b}$$

P_i and q_i are chosen to be orthogonal: $P_i\cdot q_i=0$.

To obtain the next-to-leading-order (NLO) relativistic corrections for S-wave quarkonia, we can expand the amplitude in powers of q_i, keep the quadratic terms and make the following substitution

$$q_i^\mu q_i^\nu \to \left(-g^{\mu\nu}+\frac{P_i^\mu P_i^\nu}{4E(q_i)}\right)\frac{\boldsymbol{q}_i^2}{d-1}, \tag{7}$$

where d is the dimension of spacetime. In addition, the contributions from $E(q_i)$ can be readily derived by expanding

$$E(q_i)=\sqrt{m^2+\boldsymbol{q}_i^2}=m+\frac{\boldsymbol{q}_i^2}{2m}+\mathcal{O}(\frac{\boldsymbol{q}_i^4}{m^4}), \tag{8}$$

where $m=m_b$ for $i=0$ and $m=m_c$ for $i=1,2$.

III. THE RELATIVISTIC CORRECTIONS TO THE SDCS

In this section, we calculate the SDCs of $\Upsilon\to\eta_c J/\psi$ accurate up to NLO relativistic corrections following the techniques described in last section. There are three channels which contribute, $\Upsilon\to g^*g^*g^*\to\eta_c J/\psi$, $\Upsilon\to g^*g^*\gamma^*\to\eta_c J/\psi$, and $\Upsilon\to\gamma^*\to\eta_c J/\psi$. In the subsequent three subsections, we will calculate the SDCs of the three channels separately.

A. $\Upsilon\to g^*g^*g^*\to\eta_c J/\psi$

The typical Feynman diagram is illustrated in Fig. 1(a), which is drawn by using JaxoDraw [28]. According to the Feynman rules, we write down the amplitude of

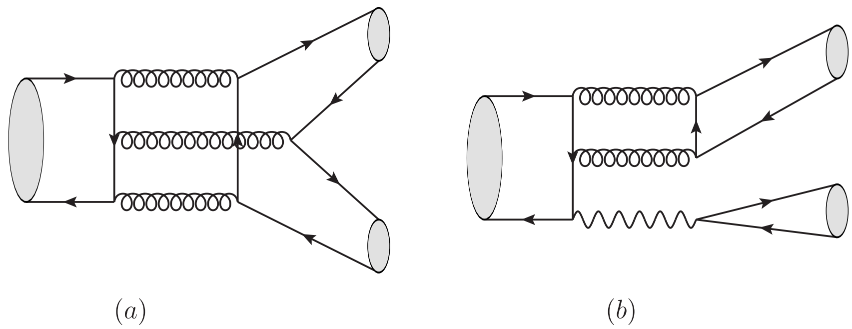

FIG. 1: The typical Feynman diagrams for $\Upsilon \to \eta_c J/\psi$, where Fig. 1(a) and Fig. 1(b) correspond to the channels $\Upsilon \to g^* g^* g^* \to \eta_c J/\psi$ and $\Upsilon \to g^* g^* \gamma^* \to \eta_c J/\psi$, respectively.

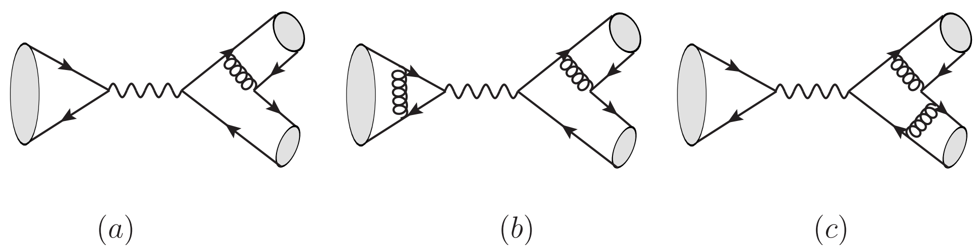

FIG. 2: The typical Feynman diagrams for $\Upsilon \to \gamma^* \to \eta_c J/\psi$. Fig. 2(a) corresponds to the tree-level diagram, and Fig. 2(b) and Fig. 2(c) correspond to the diagrams of the NLO radiative corrections from the initial state and final states, respectively.

$b\bar{b}(p_0, \bar{p}_0) \to g^* g^* g^* \to c\bar{c}(p_1, \bar{p}_1) + c\bar{c}(p_2, \bar{p}_2)$. The corresponding spin states for the heavy quark pairs are readily obtained using (5), and the relativistic corrections are calculated by employing (7) and (8). In our calculation, we use the MATHEMATICA package FEYNARTS [29] to generate the Feynman diagrams and Feynman amplitude, FEYNCALC [30] as well as FEYNCALCFORMLINK [31] to implement the algebras of Dirac trace and Lorentz indices contraction, and FIRE [32, 33] and self-written programs [34] to make the tensor reduction. After some tedious work, we finally get the amplitude, accordingly, the SDCs are obtained by removing the tensor and vectors in (1) and producing the matching factor

$$\sqrt{\frac{1}{2N_c} \frac{1}{2N_c 4E(q_1)^2} \frac{1}{2N_c 4E(q_2)^2}}. \tag{9}$$

Since there are $E(q_i)$ in (9), we should further expand them by using (8).

The SDCs are finally obtained analytically. As we will see in the next section, the channel $\Upsilon \to g^* g^* g^* \to \eta_c J/\psi$ dominates Υ decay into $\eta_c + J/\psi$; hence, it is both necessary and valuable for us to present the explicit expressions for these SDCs in the Appendix. Our LO SDCs agree with those in [16]. The numerical predictions will be given in the following section.

B. $\Upsilon \to g^* g^* \gamma^* \to \eta_c J/\psi$

The Feynman diagram is illustrated in Fig. 1(b). For this channel, the J/ψ is produced through the fragmentation of a photon. At leading order in α_s, the channel can be splitted into $\Upsilon \to \eta_c \gamma^*$ and $\gamma^* \to J/\psi$. As discussed in Ref. [35], to reduce the theoretical uncertainty, the authors apply the VMD method [36] to calculate the fragmentation of the γ^* to J/ψ in studying the process $e^+e^- \to \gamma^* \to \eta_c \gamma^* \to \eta_c J/\psi$. Similarly, here we adopt the same treatment as that in Ref. [35]. To this end, we first calculate the amplitude of the subprocess $\Upsilon \to \eta_c \gamma^*$, then multiply the amplitude with a factor

$$\frac{g_{J/\psi}\sqrt{4\pi\alpha}}{m_{J/\psi}^2}, \tag{10}$$

where the effective coupling $g_{J/\psi}$ reads [36]

$$g_{J/\psi} = \left(\frac{3m_{J/\psi}^3}{4\pi\alpha^2} \Gamma[J/\psi \to e^+e^-] \right)^{1/2}, \tag{11}$$

where $\Gamma[J/\psi \to e^+e^-]$ signifies the decay rate of the process $J/\psi \to e^+e^-$.

To get the SDCs, we apply the same techniques as in the last subsection. Finally, we obtain the analytic expressions for the amplitude, and immediately also for the SDCs. Unfortunately, the expressions of these SDCs are much cumbersome, therefore we do not attempt to present them in this paper, and merely yield the numerical predictions in the next section.

C. $\Upsilon \to \gamma^* \to \eta_c J/\psi$

The Feynman diagrams are illustrated in Fig. 2. To the accuracy considered in the current work, the amplitude of this channel can be separated into two subprocesses, $\Upsilon \to \gamma^*$ and $\gamma^* \to \eta_c J/\psi$. The NLO relativistic and radiative corrections to both subprocesses have been investigated.

For $\Upsilon \to \gamma^*$, the amplitude accurate up to $\mathcal{O}(\alpha_s v^2)$ reads [37] [2]

$$A(\Upsilon \to \gamma^*) = A^{\mathrm{Tree}}(\Upsilon \to \gamma^*) \left[1 - \frac{2\alpha_s C_F}{\pi} + \left(-\frac{1}{6} + \frac{\alpha_s C_F}{4\pi} \left(\frac{8}{9} + \frac{8}{3} \ln \frac{\mu_f^2}{m_b^2}\right) \right) \langle v^2 \rangle_\Upsilon \right], \tag{12}$$

where μ_f represents the factorization scale. For $\gamma^* \to \eta_c J/\psi$, the amplitude with the photon energy the same as that at B factories has been evaluated accurate up to $\mathcal{O}(\alpha_s v^2)$ corrections in [39, 40]. In our process, since the virtual photon is produced through Υ annihilation, it is reasonable for us to take the invariant mass of γ^* to be the mass of Υ. After some effort, we are able to obtain the expression of the amplitude for the channel $\gamma^* \to \eta_c J/\psi$ accurate up to $\mathcal{O}(\alpha_s v^2)$ corrections, and immediately yield the amplitude for $\Upsilon \to \gamma^* \to \eta_c J/\psi$ with the help of (12). Our result for $\gamma^* \to \eta_c J/\psi$ with the same energy as B factories agrees with Refs. [39, 40], where the subprocess $\gamma^* \to \eta_c J/\psi$ has been deeply investigated in [39, 40]. For details, we refer the authors to Refs. [39, 40]. In the following section, we will present the numerical predictions.

[2] It can also be read from Ref. [38] by taking $m_c = m_b$.

IV. NUMERICAL PREDICTIONS AND DISCUSSIONS

In this section, we select input parameters, and utilize the obtained analytic expressions to make numerical predictions for the SDCs as well as the decay rates.

A. Choosing the input parameters

To get the decay rates, we should calculate the squared amplitude and the phase space factor. The squared amplitude is directly derived by multiplying the amplitude in (1) with its complex conjugate. Sum over the polarizations for the final state J/ψ and average the spin states for the initial state Υ, we get

$$\frac{1}{3}\sum_{spin}|\mathcal{A}|^2 = \frac{2}{3}\left[(P_1\cdot P_2)^2 - P_1^2P_2^2\right]\times|A|^2, \tag{13}$$

where, to maintain the gauge invariance, one must take $P_1^2 = 4E^2(q_1)$, $P_2^2 = 4E^2(q_2)$ and $P_1\cdot P_2 = 2\left[E^2(q_0) - E^2(q_1) - E^2(q_2)\right]$. On the other hand, the phase space factor does not affect the gauge invariance; therefore, we use the physical masses of the quarkonia there [37]

$$\frac{1}{8\pi m_\Upsilon^2}\lambda(m_\Upsilon^2, m_{J/\psi}^2, m_{\eta_c}^2)^{1/2}, \tag{14}$$

where λ is defined via $\lambda(a,b,c)\equiv a^2+b^2+c^2-2ab-2ac-2bc$.

We now specify our choices of the parameters in our computation. We take the heavy quark pole masses as $m_c = 1.4$ GeV and $m_b = 4.6$ GeV. The masses of quarkonia are taken from Ref. [22]:

$$m_{\Upsilon(1S)} = 9.46030\ \text{GeV}, m_{\Upsilon(2S)} = 10.02326\ \text{GeV}, m_{\Upsilon(3S)} = 10.3552\ \text{GeV}, \tag{15a}$$

$$m_{\eta_c(1S)} = 2.9836\ \text{GeV}, m_{\eta_c(2S)} = 3.6394\ \text{GeV}, m_{J/\psi} = 3.096916\ \text{GeV}. \tag{15b}$$

The matrix elements for J/ψ are taken from Ref. [41],

$$\langle\mathcal{O}\rangle_{J/\psi} = 0.446\,\text{GeV}^3, \quad \langle v^2\rangle_{J/\psi} = 0.223, \tag{16}$$

which are fitted through the decay $J/\psi\to e^+e^-$ accurate through the relative order v^2, and the matrix elements of η_c are taken from Ref. [42],

$$\langle\mathcal{O}\rangle_{\eta_c(1S)} = 0.398\,\text{GeV}^3, \quad \langle v^2\rangle_{\eta_c(1S)} = 0.232, \tag{17a}$$

$$\langle\mathcal{O}\rangle_{\eta_c(2S)} = 0.202\,\text{GeV}^3, \quad \langle v^2\rangle_{\eta_c(2S)} = 0.255, \tag{17b}$$

which are fitted through the decay $\eta_c(1S)\to 2\gamma$, $\eta_c(1S)\to$ LH, and $\eta_c(2S)\to$ LH. In (16) and (17), we have used the vacuum saturation approximation [1] to relate the production matrix elements with the decay ones.

In addition, we should also choose the values of the matrix elements for Υ, which have been fitted in Refs. [20, 43]. Here, we cite the values in Ref. [43],

$$\langle\mathcal{O}\rangle_{\Upsilon(1S)} = 3.069\,\text{GeV}^3, \quad \langle v^2\rangle_{\Upsilon(1S)} = -0.009, \tag{18a}$$

$$\langle\mathcal{O}\rangle_{\Upsilon(2S)} = 1.623\,\text{GeV}^3, \quad \langle v^2\rangle_{\Upsilon(2S)} = 0.09, \tag{18b}$$

$$\langle\mathcal{O}\rangle_{\Upsilon(3S)} = 1.279\,\text{GeV}^3, \quad \langle v^2\rangle_{\Upsilon(3S)} = 0.155, \tag{18c}$$

which are fitted through the decay $\Upsilon \to e^+e^-$.

In the channel $\Upsilon \to g^*g^*\gamma^* \to \eta_c J/\psi$, we also need the decay rate for $J/\psi \to e^+e^-$, which is taken from the Ref. [22] as $\Gamma[J/\psi \to e^+e^-] = 5.55$ keV. Since the fine structure constant runs slightly with the energy scale, we uniformly take $\alpha = \frac{1}{137}$. Finally, we choose the renormalization and factorization scales to be $\mu_R = \mu_f = m_b$, and the strong coupling constant $\alpha_s(m_b) = 0.22$ [16].

B. Numerical predictions for the SDCs

In this subsection, we present the SDCs numerically by using the parameters selected in the previous subsection. As mentioned, there are three channels which contribute to SDCs; hence, we add superscripts in the SDCs to make the distinction, i.e., $c^{(i)}$, where $i = 1, 2, 3$ corresponds to the channels $\Upsilon \to g^*g^*g^* \to \eta_c J/\psi$, $\Upsilon \to g^*g^*\gamma^* \to \eta_c J/\psi$, and $\Upsilon \to \gamma^* \to \eta_c J/\psi$, respectively. Substituting the parameters into the analytic expressions of the SDCs, we obtain for $\Upsilon(1S) \to g^*g^*g^* \to \eta_c(1S)J/\psi$,

$$c_0^{(1)} = (-2.16 \times 10^{-5} - 8.90 \times 10^{-5} i)\alpha_s^3 \text{ GeV}^{-7}, \tag{19a}$$
$$c_{2,1}^{(1)} = (-5.64 - 1.83i),\ c_{2,2}^{(1)} = (-1.14 - 0.90i),\ c_{2,3}^{(1)} = (-0.99 - 0.27i); \tag{19b}$$

for $\Upsilon(1S) \to g^*g^*\gamma^* \to \eta_c(1S)J/\psi$,

$$c_0^{(2)} = (1.90 \times 10^{-4} + 7.79 \times 10^{-5} i)\alpha\alpha_s^2 \text{ GeV}^{-7}, \tag{20a}$$
$$c_{2,1}^{(2)} = (-3.34 - 0.62i),\ c_{2,2}^{(2)} = (-0.73 + 0.31i),\ c_{2,3}^{(2)} = (0.19 + 0.02i); \tag{20b}$$

and for $\Upsilon(1S) \to \gamma^* \to \eta_c(1S)J/\psi$,

$$c_0^{(3)} = (4.14 \times 10^{-5} + 6.78 \times 10^{-5}\alpha_s - 7.08 \times 10^{-5}\alpha_s i)\alpha\alpha_s \text{ GeV}^{-7}, \tag{21a}$$
$$c_{2,1}^{(3)} = (-0.17 - 0.05\alpha_s),\ c_{2,2}^{(3)} = (0.52 - 1.60\alpha_s + 2.50\alpha_s i), \tag{21b}$$
$$c_{2,3}^{(3)} = (0.35 - 0.89\alpha_s + 1.76\alpha_s i). \tag{21c}$$

To see the dependence on α and α_s, we retain these two constants in the SDCs explicitly. From (19–21), we notice some characteristics. First, $\Upsilon \to g^*g^*g^* \to \eta_c J/\psi$ is the dominant channel for the SDCs and therefore also for the decay rates. Second, the SDCs of the three channels are of quite different phase, in addition, the phases of various relativistic corrections are also distinguishing. The last is not the least, the relativistic corrections from Υ are extraordinarily large, so one may expect that this corrections will also contribute a lot to the decay rates even though the corresponding matrix element is rather small.

C. Numerical predictions for the decay rates

Combining the squared amplitude (13) with the phase space factor (14), we are able to evaluate the numerical predictions for the decay rates of various processes. All the decay rates are listed in Table. I.

In the table, we give the LO decay rates Γ_{LO} as well as the contributions from the relativistic corrections. To see it clearly, we separately list the relativistic corrections from

TABLE I: The LO and NLO relativistic corrections to decay rates of $\Upsilon \to \eta_c J/\psi$. $\Gamma_{\rm LO}$ represents the decay rate at leading order in velocity expansion; $\Gamma^{(i)}_{\rm NLO}$ with $i = 0, 1, 2$ represent the relativistic corrections from Υ, η_c, and J/ψ, respectively; $\Gamma_{\rm NLO}$ represents the total relativistic corrections.

Processes	$\Gamma_{\rm LO}$ (eV)	$\Gamma^{(0)}_{\rm NLO}$ (eV)	$\Gamma^{(1)}_{\rm NLO}$ (eV)	$\Gamma^{(2)}_{\rm NLO}$ (eV)	$\frac{\Gamma_{NLO}}{\Gamma_{\rm LO}}$
$\Upsilon(1S) \to \eta_c(1S)J/\psi$	0.41	0.03	-0.28	-0.22	-115%
$\Upsilon(1S) \to \eta_c(2S)J/\psi$	0.23	0.02	-0.17	-0.13	-122%
$\Upsilon(2S) \to \eta_c(1S)J/\psi$	0.22	-0.18	-0.15	-0.12	-204%
$\Upsilon(2S) \to \eta_c(2S)J/\psi$	0.13	-0.11	-0.10	-0.07	-211%
$\Upsilon(3S) \to \eta_c(1S)J/\psi$	0.18	-0.25	-0.12	-0.10	-263%
$\Upsilon(3S) \to \eta_c(2S)J/\psi$	0.10	-0.15	-0.08	-0.06	-270%

different sources; i.e., $\Gamma^{(i)}_{\rm NLO}$ with $i = 0, 1, 2$ originates from Υ, η_c, and J/ψ, respectively. From the table, we find that almost all the relativistic corrections are both significant and negative, except for $\Gamma^{(0)}_{\rm NLO}$ in $\Upsilon(1S)$ decay due to a tiny and negative matrix element given in (18). Both the relativistic corrections from η_c and J/ψ are negative, which is quite different with that in η_c associated with J/ψ production at B factories [44]. The discrepancy may be accounted for by distinct Feynman diagram structure.

Another distinctive feature in the table is that the relativistic corrections from $\Upsilon(2S, 3S)$ are extraordinarily large, which even outnumber the corrections from charmonia, though the NLO NRQCD matrix elements are smaller than these for charmonia. We may learn a lesson from this kind of process and the example $\Upsilon \to c\bar{c}gg$ in Refs. [20, 21]. Despite a rather small NRQCD matrix element $\langle v^2\rangle_\Upsilon$, it is not always reasonable to discard the relativistic corrections from Υ, which actually have not been considered carefully in most references. Moreover, it is also important and valuable to further determine the corresponding NRQCD matrix elements for Υ through independent approaches.

As mentioned in the introduction, $\Upsilon \to \eta_c J/\psi$ is of a similar Feynman diagram structure as $\psi \to \rho\pi$. We may expect there are also a relative large relativistic corrections for ψ in $\psi \to \rho\pi$. Nevertheless, due to both large (even huge) and negative corrections, the "$\rho - \pi$" puzzle is unlikely to be explained purely by the relativistic corrections, or at least by the NLO corrections.

By combining all the corrections, we show the ratios of the decay rates of the relativistic corrections to that at leading order in the Table. I. Our results indicate that the NLO relativistic corrections exceed the LO contributions for all the processes, and what is worse, the corrections are negative, which renders the total decay rates negative [3] and therefore unpredictable at next-to-leading order. A resummation for the large relativistic corrections may be needed [37, 38], but is, however, out of the scope of this work.

In summary, we study the NLO relativistic corrections to Υ exclusive decay into double S-wave charmonia. We calculate the contributions from three channels $\Upsilon \to g^*g^*g^* \to \eta_c + J/\psi$, $\Upsilon \to g^*g^*\gamma^* \to \eta_c + J/\psi$, and $\Upsilon \to \gamma^* \to \eta_c + J/\psi$. Our results indicate, in spite of the small $\langle v^2\rangle_\Upsilon$, the relativistic corrections from Υ are extraordinarily large, which suggests the relativistic corrections from Υ are not always negligible. Moreover, we find that all the

[3] Since decay rates are expanded in powers of v and α_s, and truncated at relative $\mathcal{O}(v^2)$, we obtain negative values.

relativistic corrections to the decay rates are both significant and negative, which indicates a poor convergence in velocity expansion for this kind of process.

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Appendix A: Analytic expressions of SDCs $c^{(1)}$ for the channel $\Upsilon \to g^* g^* g^* \to \eta_c J/\psi$

For convenience, we rewrite the SDCs as

$$c_0^{(1)} = \frac{\alpha_s^3}{6m_c^7} s_0, \quad c_0^{(1)} c_{2,1}^{(1)} = \frac{\alpha_s^3}{6m_c^7} s_1, \quad c_0^{(1)} c_{2,2}^{(1)} = \frac{\alpha_s^3}{6m_c^7} s_2, \quad c_0^{(1)} c_{2,3}^{(1)} = \frac{\alpha_s^3}{6m_c^7} s_3, \tag{A1}$$

and define

$$\begin{aligned}
\mathrm{L1} &= \mathrm{Re}\left[(-1+i\sqrt{3}\beta)\ln\frac{i(\beta-1)}{\sqrt{3}\beta-i} + (1+i\sqrt{3}\beta)\ln\frac{i(\beta+1)}{\sqrt{3}\beta+i}\right],\\
\mathrm{L2} &= \mathrm{Re}\left[\mathrm{Li}_2\frac{(-i+\sqrt{3})\beta}{\sqrt{3}\beta+i} - \mathrm{Li}_2\frac{(-i+\sqrt{3})\beta}{\sqrt{3}\beta-i}\right], \mathrm{L3} = \mathrm{Li}_2\frac{1-\beta}{2},\\
\mathrm{L4} &= \mathrm{Li}_2\frac{\beta-1}{\beta+1}, \mathrm{L5} = \mathrm{Li}_2\frac{2\beta}{\beta+1}, \mathrm{L6} = \mathrm{Li}_2\frac{\beta^2+\beta}{\beta-1}, \mathrm{L7} = \mathrm{Li}_2\frac{\beta^2+\beta}{2-6r},\\
\mathrm{L8} &= \mathrm{Li}_2\frac{\beta(\beta+1)^2}{4(1-3r)}, \mathrm{L9} = \mathrm{Li}_2\frac{\beta-\beta^2}{\beta+1}, \mathrm{L10} = \mathrm{Li}_2\frac{\beta-\beta^2}{6r-2}, \mathrm{L11} = \mathrm{Li}_2\frac{(\beta-1)^2\beta}{12r-4},\\
\mathrm{I1} &= \ln 2, \mathrm{I2} = \ln(1-\beta), \mathrm{I3} = \ln\frac{1-\beta}{1+\beta}, \mathrm{I4} = \ln(1-3r), \mathrm{I5} = \ln\frac{4(1-3r)}{(\beta+1)^2},\\
\mathrm{A1} &= \tan^{-1}(\sqrt{3}\beta),
\end{aligned} \tag{A2}$$

where $r = \frac{m_c^2}{m_b^2}$, $\beta = \sqrt{1-4r}$, and $\mathrm{Li}_2(x)$ represents the dilogarithm.

Utilizing the definitions in (A2), we present s_0, s_1, s_2, and s_3 as follows

$$
\begin{aligned}
s_0 \;=\;& \frac{20\pi(2r-1)r^3(\mathrm{I1}^2+\mathrm{I2}^2+2\mathrm{L3}+\mathrm{L4})}{27\beta^3}+\frac{5\pi(10r-3)r^3\mathrm{I3}^2}{27\beta^3}\\
&-\frac{5\pi(2r-1)r[8r^2\mathrm{I3}+\beta(2r-1)]\mathrm{I2}}{27\beta^3}-\frac{5\pi(2r-1)r(\beta+8r^2\mathrm{I2}-8r^2\mathrm{I3}-2\beta r)\mathrm{I1}}{27\beta^3}\\
&+\frac{5\pi r\left[\beta-8(2\mathrm{I4}+i\pi+4)r^3+4(\beta+i\pi+4)r^2-2(2\beta+3)r+1\right]\mathrm{I3}}{54\beta^3}\\
&+\frac{40\pi r^4}{27\beta^3}\left(\frac{2\pi}{3}\mathrm{A1}+2\mathrm{L2}+2\mathrm{L5}+\mathrm{L6}+\mathrm{L7}-\mathrm{L8}-\mathrm{L9}-\mathrm{L10}+\mathrm{L11}\right)\\
&-\frac{5\pi r^2\left[3\beta^3+2(2\sqrt{3}-3i)\pi\beta r+\pi^2 r(2r-1)\right]}{81\beta^3},
\end{aligned}
\tag{A3}
$$

$$
\begin{aligned}
s_1 \;=\;& \frac{40\pi r^5\mathrm{L1}}{27(3r-1)\beta^5}+\frac{40\pi r^4(\mathrm{I4}-\mathrm{I5})}{27\beta^4}+\frac{10\pi(104r^2-8r-9)r^3}{81\beta^5}(\frac{2\pi}{3}\mathrm{A1}+2\mathrm{L2}+2\mathrm{L5}+\mathrm{L6}+\mathrm{L7}\\
&-\mathrm{L8}-\mathrm{L9}-\mathrm{L10}+\mathrm{L11})+\frac{10\pi(304r^2-206r+37)r^3}{81\beta^5}(\mathrm{I1}^2+\mathrm{I2}^2+2\mathrm{L3}+\mathrm{L4})\\
&+\frac{5\pi(1120r^2-634r+93)r^3\mathrm{I3}^2}{162\beta^5}\\
&-\frac{5\pi}{972(12r^2-7r+1)^2\beta}\Big\{2\pi^2(1-3r)^2(304r^2-206r+37)r^3+4\pi\Big[8(-405i+181\sqrt{3})r^3\\
&+(3213i-1532\sqrt{3})r^2+6(-177i+88\sqrt{3})r-60\sqrt{3}+117i\Big]\beta r^3+(-14976r^6+12888r^5\\
&-1162r^4-2593r^3+1286r^2-249r+18)\beta\Big\}+\frac{5\pi}{486\beta^5 r}\Big[24(304r^2-206r+37)\\
&r^4(\mathrm{I1I3}-\mathrm{I1I2}-\mathrm{I2I3})+(480r^5-44r^4-200r^3+235r^2-84r+9)(\mathrm{I1}-\mathrm{I2})\beta\Big]\\
&+\frac{5\pi\mathrm{I3}}{486\beta^5 r}\Big\{\frac{\beta^3}{(12r^2-7r+1)^2(\beta+1)}\Big[3456(40+19i\pi)r^9-144i(\pi(114\beta+727)\\
&-48i(7\beta+33))r^8+4(14598\beta+3i\pi(1839\beta+5585)+34606)r^7+(-20974\beta\\
&-6i\pi(1873\beta+3585)-25754)r^6+(-5731\beta+24i\pi(107\beta+144)-18253)r^5\\
&+2(4749\beta-111i\pi(\beta+1)+8515)r^4-2(2353\beta+3431)r^3+(1225\beta+1537)r^2\\
&-3(55\beta+61)r+9(\beta+1)\Big]-12r^4(104r^2-8r-9)\mathrm{I4}\Big\},
\end{aligned}
\tag{A4}
$$

$$
\begin{aligned}
s_2 \; = \; & \frac{10\pi(13r-1)r^3}{81\beta^5}\left(\frac{2\pi}{3}\mathrm{A1}+2\mathrm{L2}+2\mathrm{L5}+\mathrm{L6}+\mathrm{L7}-\mathrm{L8}-\mathrm{L9}-\mathrm{L10}+\mathrm{L11}\right) \\
& +\frac{20\pi(40r^3-15r+2)r^3(\mathrm{I1}^2+\mathrm{I2}^2+2\mathrm{L3}+\mathrm{L4})}{81\beta^5}+\frac{5\pi(120r^3-32r+5)r^3\mathrm{I3}^2}{81\beta^5} \\
& +\frac{5\pi}{486\beta^5}\Big[48(40r^3-15r+2)r^3(\mathrm{I1I3}-\mathrm{I1I2}-\mathrm{I2I3})+\beta(288r^5-352r^4-232r^3 \\
& +224r^2-48r+3)(\mathrm{I1}-\mathrm{I2})\Big]-\frac{20\pi r^4(\mathrm{I4}-\mathrm{I5})}{27\beta^4}-\frac{20\pi r^5\mathrm{L1}}{27\beta^5(3r-1)} \\
& +\frac{5\pi\mathrm{I3}}{486\beta^5}\Big\{-12(13r-1)r^3\mathrm{I4}-\frac{\beta^3}{(\beta+1)(12r^2-7r+1)^2}\Big[-3(\beta+1) \\
& -384i(45\pi-236i)r^9+96(290\beta+15i\pi(3\beta+11)+1402)r^8 \\
& +16(-2523\beta-15i\pi(12\beta-7)-5003)r^7+4(3842\beta-3i\pi(95\beta+527)+2110)r^6 \\
& +4(1742\beta+324i\pi(\beta+2)+5075)r^5+(-8805\beta-12i\pi(27\beta+35)-14809)r^4 \\
& +(3583\beta+24i\pi(\beta+1)+4913)r^3-2(367\beta+439)r^2+(75\beta+81)r\Big]\Big\} \\
& -\frac{5\pi r}{8748\beta(12r^2-7r+1)^2}\Big[36\pi^2(1-3r)^2(40r^3-15r+2)r^2 \\
& +4\pi\beta(216(70\sqrt{3}-93i)r^4-4(4036\sqrt{3}-4401i)r^3+(6468\sqrt{3}-5931i)r^2 \\
& -18(68\sqrt{3}-59i)r+100\sqrt{3}-99i)r^2-9\beta(10560r^6-9568r^5+648r^4+1983r^3 \\
& -824r^2+123r-6)\Big],
\end{aligned} \tag{A5}
$$

and

$$
\begin{aligned}
s_3 \;=\; & -\frac{100\pi r^5\mathrm{L1}}{27(3r-1)\beta^5}-\frac{100\pi r^4(\mathrm{I4}-\mathrm{I5})}{27\beta^4}+\frac{10\pi(72r^2+31r-1)r^3}{81\beta^5}\bigg(\frac{2\pi}{3}\mathrm{A1}+2\mathrm{L2}+2\mathrm{L5}+\mathrm{L6} \\
& +\mathrm{L7}-\mathrm{L8}-\mathrm{L9}-\mathrm{L10}+\mathrm{L11}\bigg) \\
& +\frac{10\pi(400r^3-24r^2-140r+19)r^3(\mathrm{I1}^2+\mathrm{I2}^2+2\mathrm{L3}+\mathrm{L4})}{81\beta^5} \\
& +\frac{5\pi(1200r^3+72r^2-358r+55)r^3\mathrm{I3}^2}{162\beta^5}-\frac{5\pi}{8748(12r^2-7r+1)^2\beta}\bigg[18\pi^2(1-3r)^2 \\
& (400r^3-24r^2-140r+19)r^3+8\pi(540(-93i+70\sqrt{3})r^4+(43524i-40036\sqrt{3})r^3 \\
& +3(-4794i+5291\sqrt{3})r^2-90(-28i+33\sqrt{3})r+241\sqrt{3}-234i)\beta r^3-9(52800r^7 \\
& -57632r^6+22872r^5-7217r^4+4017r^3-1565r^2+279r-18)\beta\bigg] \\
& +\frac{5\pi}{486\beta^5 r}\bigg[24(400r^3-24r^2-140r+19)r^4(\mathrm{I1I3}-\mathrm{I2I3}-\mathrm{I1I2})+(1440r^6-1904r^5 \\
& -4r^4-100r^3+307r^2-99r+9)(\mathrm{I1}-\mathrm{I2})\beta\bigg] \\
& +\frac{5\pi\mathrm{I3}}{486\beta^5 r}\bigg\{\frac{\beta^3}{(12r^2-7r+1)^2(\beta+1)}\bigg[1920(236+45i\pi)r^{10} \\
& -96i(3\pi(75\beta+293)-2i(725\beta+3421))r^9+16(12525\beta+3i\pi(327\beta-31)+25513)r^8 \\
& +8i(537\pi\beta+12797i\beta+3498\pi+16349i)r^7+2(11339\beta-3i\pi(987\beta+2003)+15363)r^6 \\
& +2i(762\pi\beta+5240i\beta+990\pi+12631i)r^5+(11041\beta-114i\pi(\beta+1)+20095)r^4 \\
& -4(1411\beta+2046)r^3+2(716\beta+887)r^2-18(10\beta+11)r+9(\beta+1)\bigg] \\
& -12r^4(72r^2+31r-1)\mathrm{I4}\bigg\}. \qquad \text{(A6)}
\end{aligned}
$$

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